PIEZOELECTRIC TECHNOLOGY PRIMER

James R. Phillips
Sr. Member of Technical Staff
CTS Wireless Components
4800 Alameda Blvd. N.E.
Albuquerque, New Mexico 87113

Piezoelectricity

The piezoelectric effect is a property that exists in many materials. The name is made up of two parts; piezo, which is derived from the Greek work for pressure, and electric from electricity. The rough translation is, therefore, pressure - electric effect. In a piezoelectric material, the application of a force or stress results in the development of a charge in the material. This is known as the direct piezoelectric effect. Conversely, the application of a charge to the same material will result in a change in mechanical dimensions or strain. This is known as the indirect piezoelectric effect.

Several ceramic materials have been described as exhibiting a piezoelectric effect. These include lead-zirconate-titanate (PZT), lead-titanate ($PbTiO_2$), lead-zirconate ($PbZrO_3$), and barium-titanate ($BaTiO_3$). These ceramics are not actually piezoelectric but rather exhibit a polarized electrostrictive effect. A material must be formed as a single crystal to be truly piezoelectric. Ceramic is a multi crystalline structure made up of large numbers of randomly orientated crystal grains. The random orientation of the grains results in a net cancelation of the effect. The ceramic must be polarized to align a majority of the individual grain effects. The term piezoelectric has become interchangeable with polarized electrostrictive effect in most literature.

Piezoelectric Effect

It is best to start with an understanding of common dielectric materials in order to understand the piezoelectric effect. The defining equations for high permittivity dielectrics are:

$$C = \frac{K \varepsilon_r A}{t} = \frac{\varepsilon_O \varepsilon_r A}{t} = \frac{\varepsilon A}{t}$$

and

$$Q = C V \longrightarrow Q = \frac{\varepsilon A V}{t}$$

where:
C = capacitance
A = capacitor plate area
ε_r = relative dielectric constant
ε_O = dielectric constant of air = 8.85×10^{-12} farads / meter
ε = dielectric constant
V = voltage
t = thickness or plate separation
Q = charge

In addition, we can define electric displacement, D, as charge density or the ratio of charge to the area of the capacitor:

$$D = \frac{Q}{A} = \frac{\varepsilon V}{t}$$

and further define the electric field as:

$$E = \frac{V}{t} \qquad \text{or} \qquad D = \varepsilon E$$

These equations are true for all isotropic dielectrics. Piezoelectric ceramic materials are isotropic in the unpolarized state, but they become anisotropic in the poled state. In anisotropic materials, both the electric field and electric displacement must be represented as vectors with three dimensions in a fashion similar to the mechanical force vector. This is a direct result of the dependency of the ratio of dielectric displacement, D, to electric field, E, upon the orientation of the capacitor plate to the crystal (or poled ceramic) axes. This means that the general equation for electric displacement can be written as a state variable equation:

$$D_i = \varepsilon_{ij} E_j$$

The electric displacement is always parallel to the electric field, thus each electric displacement vector, D_i, is equal to the sum of the field vector, E_j, multiplied by its corresponding dielectric constant, ε_{ij}:

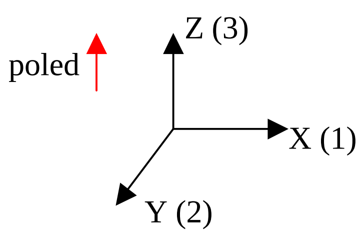
$$D_1 = \varepsilon_{11} E_1 + \varepsilon_{12} E_2 + \varepsilon_{13} E_3$$
$$D_2 = \varepsilon_{21} E_1 + \varepsilon_{22} E_2 + \varepsilon_{23} E_3$$
$$D_3 = \varepsilon_{31} E_1 + \varepsilon_{32} E_2 + \varepsilon_{33} E_3$$

Fortunately, the majority of the dielectric constants for piezoelectric ceramics (as opposed to single crystal piezoelectric materials) are zero. The only non-zero terms are:

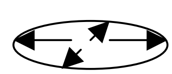
$$\varepsilon_{11} = \varepsilon_{22} \ , \ \varepsilon_{33}$$

Axis Nomenclature

The piezoelectric effect, as stated previously, relates mechanical effects to electrical effects. These effects, as shown above, are highly dependent upon their orientation to the poled axis. It is, therefore, essential to maintain a constant axis numbering scheme.



#	Axis
1	X
2	Y
3	Z (poled)
4	Shear Around X
5	Shear Around Y
6	Shear Around Z
P	Radial Vibration



for electro-mechanical constants:

d_{ab}, a = electrical direction; b = mechanical direction

Electrical - Mechanical Analogies

Piezoelectric devices work as both electrical and mechanical elements. There are several electrical - mechanical analogies that are used in designing modeling the devices.

Electrical Unit		Mechanical Unit	
e	Voltage (Volts)	f	Force (Newtons)
i	Current (Amps)	v	Velocity (Meters / Second)
Q	Charge (Coulombs)	s	Displacement (Meters)
C	Capacitance (farads)	C_M	Compliance (Meters / Newton)
L	Inductance (henrys)	M	Mass (Kg)
Z	Impedance	Z_M	Mechanical Impedance

$$i = \frac{dQ}{dt} \qquad v = \frac{ds}{dt}$$

$$e = L\frac{di}{dt} = L\frac{d^2Q}{dt^2} \qquad f = M\frac{dv}{dt} = M\frac{d^2s}{dt^2}$$

Coupling

Coupling is a key constant used to evaluate the "quality" of an electro-mechanical material. This constant represents the efficiency of energy conversion from electrical to mechanical or mechanical to electrical.

$$k^2 = \frac{\text{Mechanical Energy Converted to Electrical Charge}}{\text{Mechanical Energy Input}}$$

or

$$k^2 = \frac{\text{Electrical Energy Converted to Mechanical Displacement}}{\text{Electrical Energy Input}}$$

Electrical, Mechanical Property Changes With Load

Piezoelectric materials exhibit the somewhat unique effect that the dielectric constant varies with mechanical load and the Young's modulus varies with electrical load.

Dielectric Constant

$$\varepsilon_{r\ FREE}\,(1 - k^2) = \varepsilon_{r\ CLAMPED}$$

This means that the dielectric "constant" of the material reduces with mechanical load. Here "Free" stands for a state when the material is able to change dimensions with applied field. "Clamped" refers to either a condition where the material is physically clamped or is driven at a frequency high enough above mechanical resonance that the device can't respond to the changing E field.

Elastic Modulus (Young's Modules)

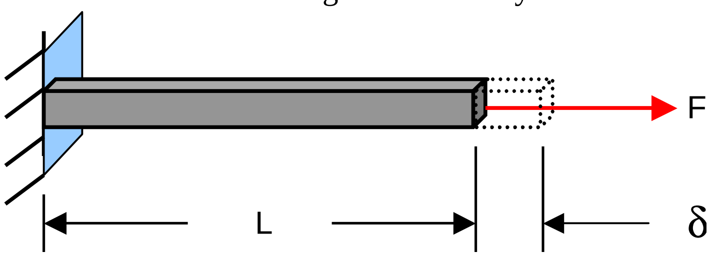
$$Y_{OPEN}(1-k^2) = Y_{SHORT}$$

This means that the mechanical "stiffness" of the material reduces when the output is electrically shorted. This is important in that both the mechanical Q_M and resonate frequency will change with load. This is also the property that is used in the variable dampening applications.

Elasticity

All materials, regardless of their relative hardness, follow the fundamental law of elasticity. The elastic properties of the piezoelectric material control how well it will work in a particular application. The first concepts, which need to be defined, are stress and strain.

for a given bar of any material:



F = Applied Force
A = Area To Which Force Is Applied
δ = Elongation

$$\text{Stress} = \sigma = F / A$$
$$\text{Strain} = \lambda = \delta / L$$

The relationship between stress and strain is Hooke's Law which states that, within the elastic limits of the material, strain is proportional to stress.

$$\lambda = s\,\sigma$$

or, for an anisotropic material

$$\lambda_i = S_{ij}\,\sigma_j$$

Note: The constant relating stress and strain is the modulus of elasticity or Young's modulus and is often represented by S, E or Y.

Piezoelectric Equation

It has been previously shown that when a voltage is applied across a capacitor made of normal dielectric material, a charge results on the plates or electrodes of the capacitor. Charge can also be produced on the electrodes of a capacitor made of a piezoelectric material by the application of stress. This is known as the Direct Piezoelectric Effect.

Conversely, the application of a field to the material will result in strain. This is known as the Inverse Piezoelectric Effect. The equation, which defines this relationship, is the piezoelectric equation.

$$D_i = d_{ij}\,\sigma_j$$

where:

$D_i \equiv$ Electric Displacement (or Charge Density)

$d_{ij} \equiv$ Piezoelectric Modulus, the ratio of strain to applied field or charge density to applied mechanical stress

Stated differently, d measures charge caused by a given force or deflection caused by a given voltage. We can, therefore, also use this to define the piezoelectric equation in terms of field and strain.

$$Di = \frac{\sigma_j\,\lambda_i}{E_j}$$

Earlier, electric displacement was defined as

$$Di = \varepsilon_{ij}\,E_j$$

therefore,

$$e_{ij}\,E_j = d_{ij}\,\sigma$$

and

$$Ej = \frac{d_{ij}}{\varepsilon_{ij}}\,\sigma_j$$

which results in a new constant

$$gij = \frac{d_{ij}}{\varepsilon_{ij}}$$

This constant is known as the piezoelectric constant and is equal to the open circuit field developed per unit of applied stress or as the strain developed per unit of applied charge density or electric displacement. The constant can then be written as:

$$g = \frac{\text{field}}{\text{stress}} = \frac{\text{volts / meter}}{\text{newtons / meter}^2} = \frac{\Delta L / L}{\varepsilon\, V / t}$$

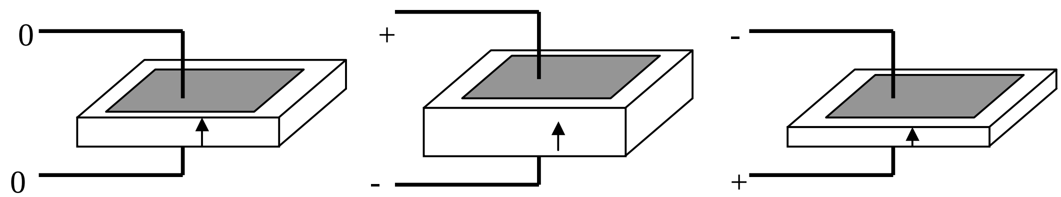
Fortunately, many of the constants in the formulas above are equal to zero for PZT piezoelectric ceramics. The non-zero constants are:

$$s_{11} = s_{22},\ s_{33},\ s_{12},\ s_{13} = s_{23},\ s_{44},\ s_{66} = 2\,(s_{11} - s_{12})$$

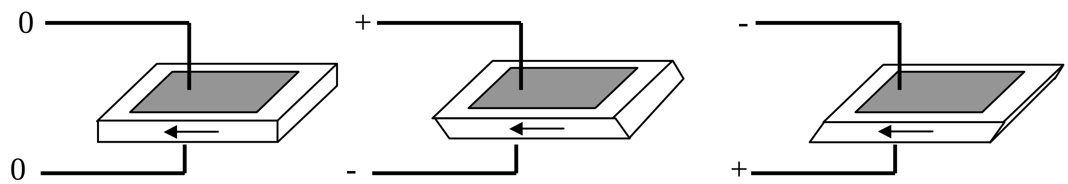
$$d_{31} = d_{32},\ d_{33},\ d_{15} = d_{24}$$

Basic Piezoelectric Modes

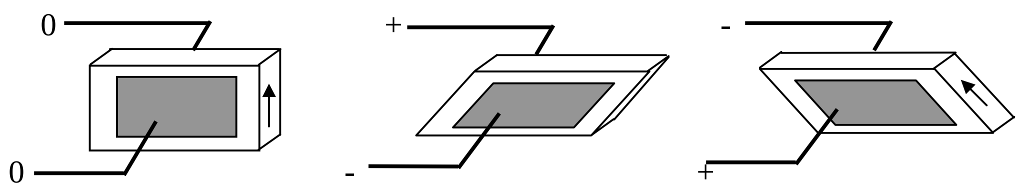
Thickness Expansion



Thickness Shear



Face Shear



↑ Polarization Direction

Poling

Piezoelectric ceramic materials, as stated earier, are not piezoelectric until the random ferroelectric domains are aligned. This alignment is accomplished through a process known as "poling". Poling consists of inducing a D.C. voltage across the material. The ferroelectric domains align to the induced field resulting in a net piezoelectric effect. It should be noted that not all the domains become exactly aligned. Some of the domains only partially align and some do not align at all. The number of domains that align depends upon the poling voltage, temperature, and the time the voltage is held on the material. During poling the material permanetly increases in dimension between the poling electrodes and decreases in dimensions parallel to the electrodes. The material can be de-poled by reversing the poling voltage, increasing the temperature beyond the materials Currie point, or by inducing a large mechanical stress.

Post Poling

Applied Voltage:

Voltage applied to the electrodes at the same polarity as the original poling voltage results in a further increase in dimension between the electrodes and decreases the dimensions parallel to the electrodes. Applying a voltage to the electrodes in an opposite direction decreases the dimension between the electrodes and increases the dimensions parallel to the electrodes.

Applied Force:

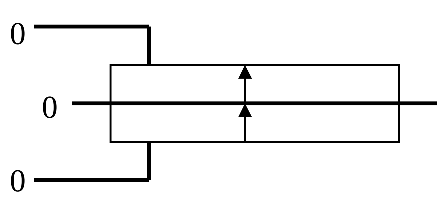
Applying a compressive force in the direction of poling (perpendicular to the poling electrodes) or a tensile force parallel to the poling direction results in a voltage generated on the electrodes which has the same polarity as the original poling voltage. A tensile force applied perpendicular to the electrodes or a compressive force applied parallel to the electrodes results in a voltage of opposite polarity.

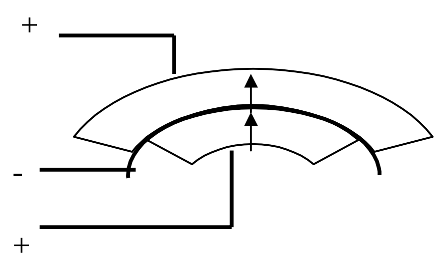
Shear:

Removing the poling electrodes and applying a field perpendicular to the poling direction on a new set of electrodes will result in mechanical shear. Physically shearing the ceramic will produce a voltage on the new electrodes.

Piezoelectric Benders

Piezoelectric benders are often used to create actuators with large displacement capabilities. The bender works in a mode which is very similar to the action of a bimetallic spring. Two separate bars or wafers of piezoelectric material are metallized and poled in the thickness expansion mode. They are then assembled in a + -+ - stack and mechanically bonded. In some cases, a thin membrane is placed between the two wafers. The outer electrodes are connected together and a field is applied between the inner and outer electrodes. The result is that for one wafer the field is in the same direction as the poling voltage while the other is opposite to the poling direction, This means that one wafer is increasing in thickness and decreasing in length while the other wafer is decreasing in thickness and increasing in length, resulting in a bending moment.





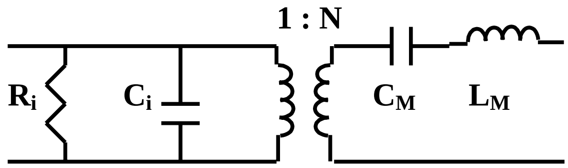
Loss

There are two sources for loss in a piezoelectric device. One is mechanical, the other is electrical.

Mechanical Loss: $Q_M = \dfrac{\text{Mechanical Stiffness Reactance or Mass Reactance}}{\text{Mechanical Resistance}}$

Electrical Loss: $\tan \delta = \dfrac{\text{Effective Series Resistance}}{\text{Effective Series Reactance}}$

Simplified Piezoelectric Element Equivalent Circuit



R_i = Electrical Resistance

C_i = Input Capacitance = $\dfrac{\varepsilon_O \varepsilon_r A}{t}$

ε_O = 8.85 X 10-12 farads / meter
A = Electrode Area
t = Dielectric Thickness

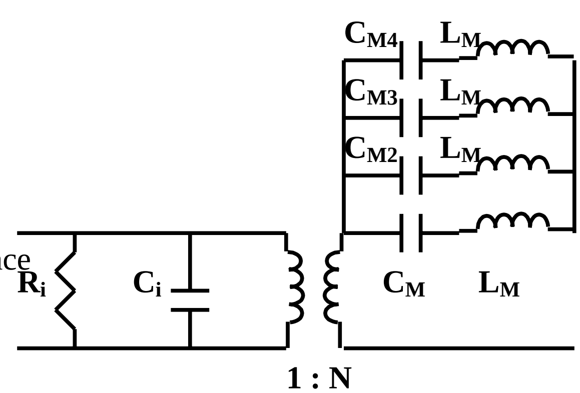
L_M = Mass (Kg)

C_M = Mechanical Compliance = 1 / Spring Rate (M / N)

N = Electro-mechanical Linear Transducer Ratio (newtons/volt or coulomb/meter)

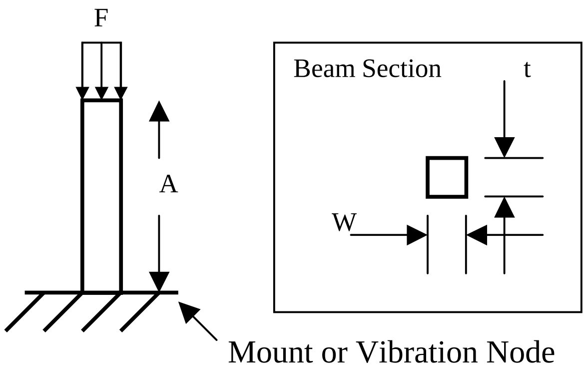
This model has been simplified and it is missing several factors. It is only valid up to and slightly beyond resonance. The first major problem with the model is related to the mechanical compliance (C_M). Compliance is a function of mounting, shape, deformation mode (thickness, free bend, cantilever, etc.) and modulus of elasticity. The modulus of elasticity is, however, anisotropic and it varies with electrical load. The second issue is that the resistance due to mechanical Q_M has been left out. Finally, there are many resonant modes in the transformers, each of which has its own C_M as shown below.

Mechanical Compliance



Mechanical compliance, which is the inverse of spring constant, is a function of the shape, mounting method, modulus and type of load. Some simple examples are shown below.

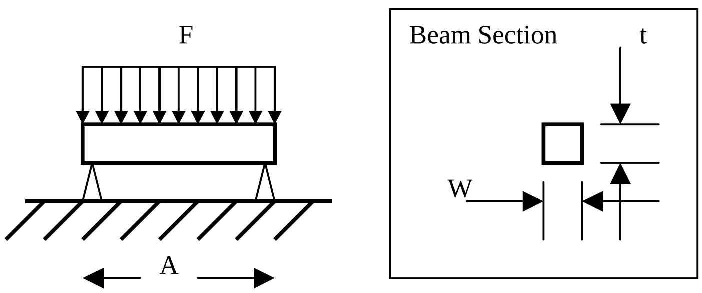
Simple Beam - Uniform End Load



$$C_M = \frac{A}{AY_{ij}} = \frac{A}{W\ t}$$

$$L_M = \rho\ A\ W\ t$$

Simple Beam - Uniform Load - End Mounts



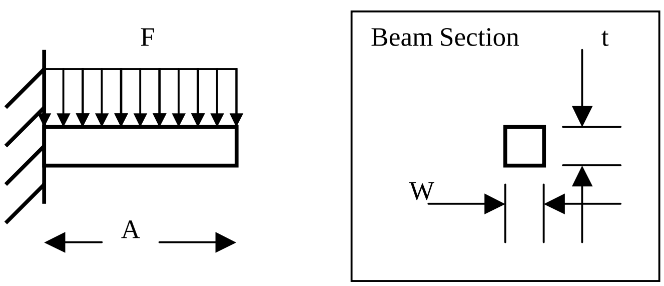
$$C_M = \frac{5\ A^3}{384\ Y_{ij}\ I}$$

I = Moment of Inertia

$$= \frac{W\ t3}{12}$$

$$C_M = \frac{5\ A^3}{32\ W\ t^3}$$

Simple Beam - Uniform Load - Cantilever Mount



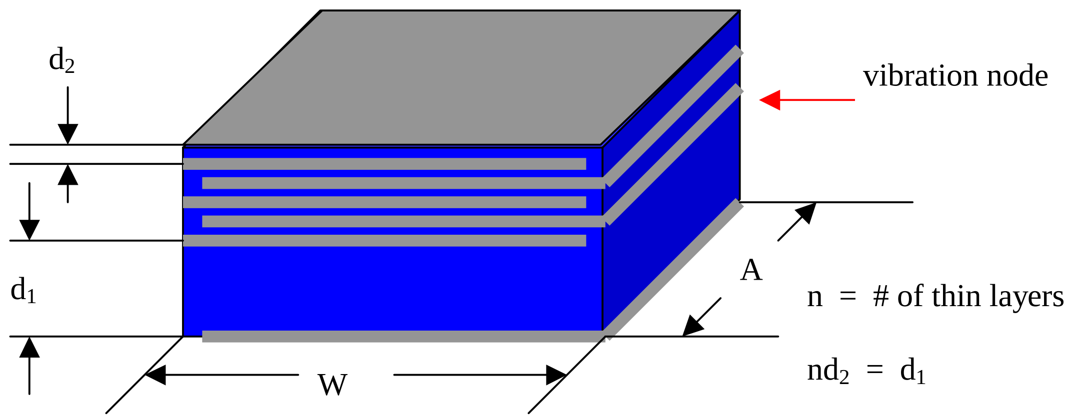
$$C_M = \frac{A^3}{8Y_{ij}I}$$

$$= \frac{3\ A^3}{2\ Y_{ij}\ W\ t^3}$$

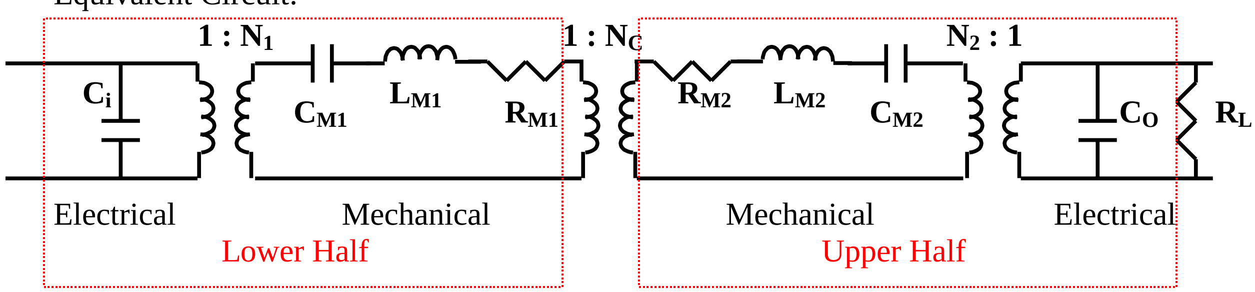
The various elements that have been explained can now be combined into the design of a complete piezoelectric device. The simple piezoelectric stack transformer will be used to demonstrate the way they are combined to create a functional model.

Simple Stack Piezoelectric Transformer

The piezoelectric transformer acts as an ideal tool to explain the modeling of piezoelectric devices in that it utilizes both the direct and indirect piezoelectric effects. The transformer operates by first converting electrical energy into mechanical energy in one half of the transformer. This energy is in the form of a vibration at the acoustic resonance of the device. The mechanical energy produced is then mechanically coupled into the second half of the transformer. The second half of the transformer then reconverts the mechanical energy into electrical energy. The figure below shows the basic layout of a stack transformer. The transformer is driven across the lower half (dimension d_1) resulting in a thickness mode vibration. This vibration is coupled into the upper half and the output voltage is taken across the thinner dimension d_2.



Equivalent Circuit:



The equivalent circuit model for the transformer (shown above) can be thought of as two piezoelectric elements that are assembled back to back. These devices are connected together by an ideal transformer representing the mechanical coupling between the upper and lower halves. The input resistance, R_i, and the output resistance, R_O, are generally very large and have been left out in this model. The resistor RL represents the applied load. Determining the values of the various components can be calculated as shown previously.

Input / Output Capacitance:

$$C_i = \varepsilon_O \varepsilon_r \frac{\text{Input Area}}{\text{Input Thickness}} = \varepsilon_O \varepsilon_r \frac{A\,W}{d_1}$$

similarly,

$$C_O = \varepsilon_O \varepsilon_r \frac{\text{Output Area}}{\text{Output Thickness}} = \varepsilon_O \varepsilon_r \frac{n A W}{d_2}$$

Mechanical Compliance:

The mechanical compliance, C_M, can be represented by a simple beam subjected to a uniform axial load. This is because the thickness expansion mode will apply uniform stress across the surface. It should be noted that the beam length is measured with respect to the vibration node. The vibration node is used as this is the surface which does not move at resonance and can, therefore, be thought of as a fixed mounting surface.

$$C_M = \frac{\text{Beam Length}}{\text{Beam Area } Y_{33}}$$

$$C_{M1} = \frac{d_1}{AW\, Y_{33}}$$

$$C_{M2} = \frac{d_2}{AW\, Y_{33}}$$

Note: Even if $nd_2 \neq d_1$ the vibration node will still be located in the mechanical center of the transformer.

Mass:

$$L_{M1} = \rho\, A\, W\, d_1$$

$$L_{M2} = \rho\, A\, W\, nd_2 = \rho\, A\, W\, d_1$$

Resistance:

The resistances in the model are a function of the mechanical Q_M and Q of the material at resonance and will be calculated later.

Ideal Transformer Ratio:

The transformer ratio, N_1, can be thought of as the ratio of electrical energy input to the resulting mechanical energy output. This term will then take the form of newtons per volt and can be derived form the piezoelectric constant, g.

as before:

$$g = \frac{\text{Electric Field}}{\text{Stress}} = \frac{\text{Volts / Meter}}{\text{Newtons / Meter}^2}$$

therefore:

$$\frac{1}{g} = \frac{n / m}{V / m}$$

$$N_1 = \frac{1}{g} \frac{\text{Area Of Applied Force}}{\text{Length Of Generated Field}}$$

or

$$N_1 = \frac{A\ W}{g_{33}\ d_1}$$

The output section converts mechanical energy back to electrical energy and the ratio would normal be calculated in an inverse fashion to N_1. In the model, however, the transformer ratio is shown as N_2 : 1. This results in a calculation for N_2 that is identical to the calculation of N_1.

$$N_2 = \frac{1}{g} \frac{\text{Area Of Applied Force}}{\text{Length Of Generated Field}}$$

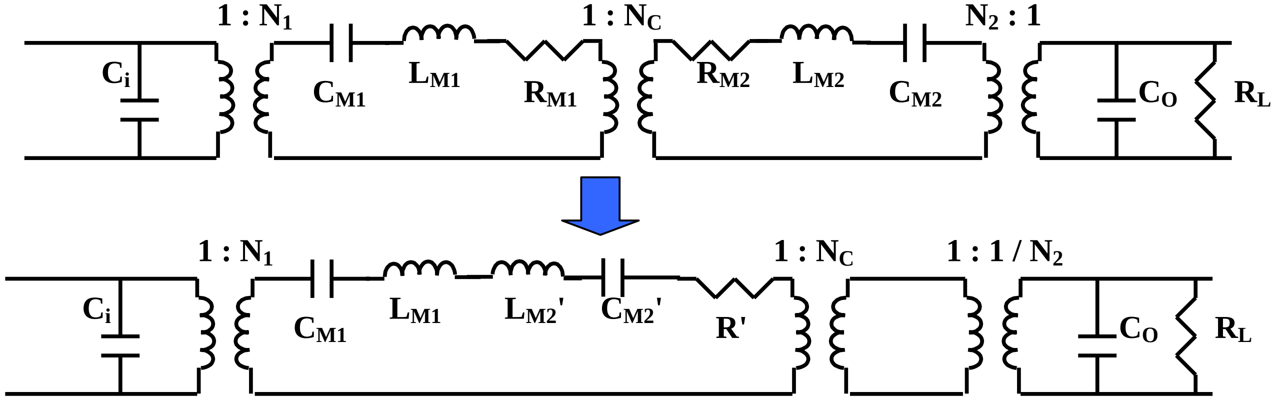
or

$$N_2 = \frac{A\ W}{g_{33}\ d_2}$$

The transformer 1 : N_C, represents the mechanical coupling between the two halves of the transformer. The stack transformer is tightly coupled and the directions of stress are the same in both halves. This results in $N_C \cong 1$.

Model Simplification:

The response of the transformer can be calculated from this model, but it is possible to simplify the model through a series of simple network conversion and end up in an equivalent circuit whose form is the same as that of a standard magnetic transformer.



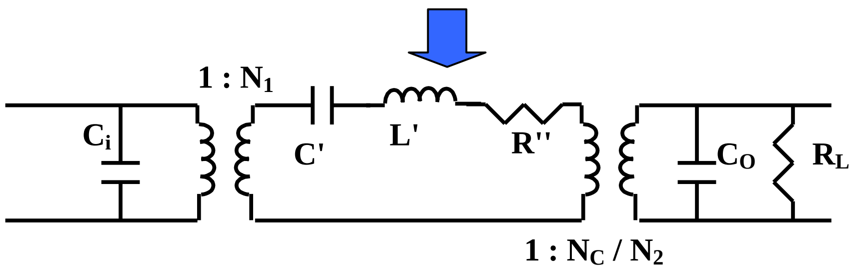
where, due to translation through the transformer,

$$C_{M2}' = N_C^2\, C_{M2} \quad \text{and} \quad L_{M2}' = L_{M2} / N_C^2$$

but $N_C^2 \cong 1$, therefore

$$C_{M2}' = C_{M2} = C_{M1} \quad \text{and} \quad L_{M2}' = L_{M2} = L_{M1}$$

which allows the next level of simplification

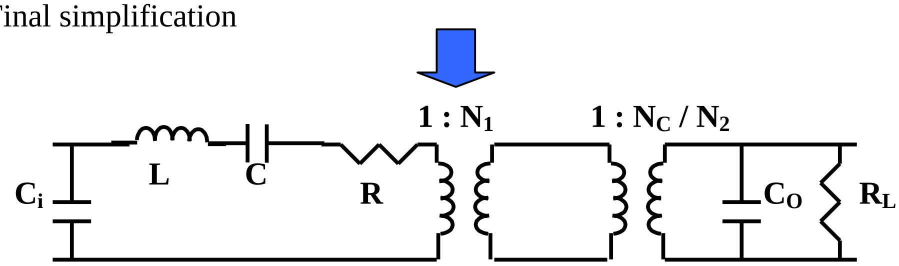


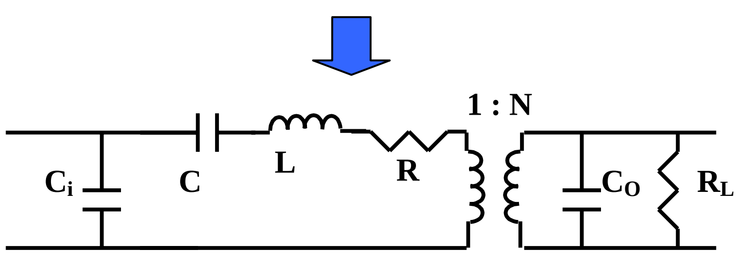
here

$$L' = L_{M1} + L_{M2}' = 2\, L_1 = 2\, \rho\, A\, W\, d_1$$

$$C' = \frac{(C_{M1}\, C_{M2}')}{(C_{M1} + C_{M2}')} = \frac{C_{M1}^2}{2C_{M1}} = \frac{C_{M1}}{2} = \frac{d_1}{2\, A\, W\, Y_{33}}$$

Final simplification





where

$$C = C'\, N_1^2 \quad \text{and} \quad L = L' / N_1^2$$

and, from before

$$N_1 = \frac{A\ W}{g_{33} d_1}$$

therefore

$$C = \frac{d1}{2WLY_{33}} \frac{A^2\ W^2}{g_{33}^2\ d_1^2} = \frac{A\ W}{2Y_{33} g_{33}^2 d_1}$$

$$L = 2\rho\ A\ W\ d_1\ \frac{g_{33}^2\ d_1^2}{A^2\ W^2} = \frac{2\rho\ g_{33}^2\ d_1^2}{AW}$$

$$N = \frac{N_1\ N_C}{N_2} = \frac{A\ W}{g_{33} d_1} \frac{g_{33} d_2}{A\ W} = \frac{d_2}{d_1}$$

The last value we need to calculate is the motional resistance. This value is based upon the mechanical QM of the material and the acoustic resonant frequency.

Resonate Frequency

$$\omega_O = 1 / \sqrt{L\ C}$$

$$= \frac{1}{\sqrt{\frac{2\rho\ d_1\ g_{33}^2}{A\ W} \frac{A\ W}{2Y_{33} g_{33}^2\ d1}}}$$

$$= \frac{1}{\sqrt{\frac{\rho\ d_1^2}{Y33}}} = \frac{1}{d_1 \sqrt{\frac{\rho}{Y_{33}}}}$$

$$c_{PZT} \equiv \text{speed of sound in PZT} = \sqrt{Y\ /\ \rho}$$

therefore

$$\omega_O = c_{PZT}\ /\ d_1$$

The equation shown above states that the resonant frequency is equal to the speed of sound in the material divided by the acoustic length of the device. This is the definition of acoustic resonance and acts as a good check of the model. The final derivation is the value of resistance.

$$Q_M \equiv 1/\ \omega_O R C$$

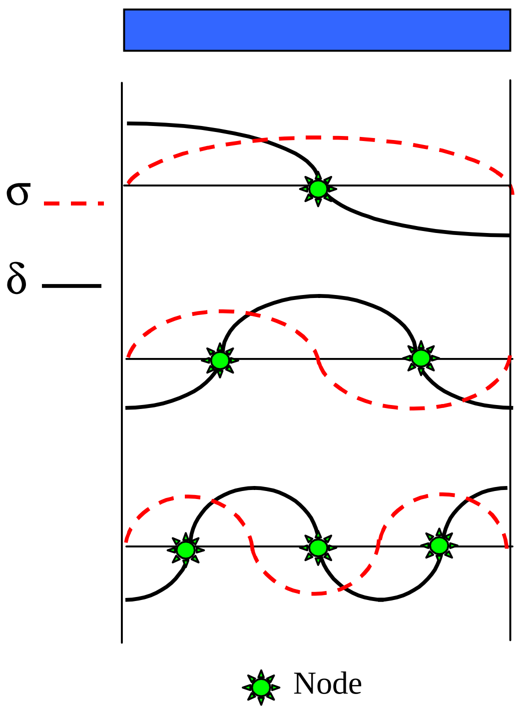
or

$$R = 1 / \omega_O Q_M C$$

$$R = \frac{d_1 \sqrt{\rho / Y_{33}}}{Q_M} \; \frac{2 Y_{33} g_{33}^2 d_1}{A W} = \frac{2 d_1^2 g_{33}^2 \sqrt{\rho Y_{33}}}{Q_M A W}$$

Note: C_M and R are both functions of Y_{33} and Y_{33} is a function of R_L

It should be noted that the model is only valid for transformers driven at or near their fundamental resonate frequencies. This is because the initial mechanical model assumed a single vibration node located at the center of the stack. which is only true when the transformer is driven at fundamental resonance. There are more nodes when the transformer is driven at harmonic frequencies.



Frequency	Wave Length
ω_O	$\lambda / 2$
$2\omega_O$	λ
$3\omega_O$	$3\lambda / 2$

Note: Stress is 90^o out of phase from displacement

There are no fixed nodes at frequencies other than resonance. This means that the transformer must be designed with the resonate mode in mind or phase cancellations will occur and there will be little or no voltage gain. It is often difficult to understand the concept of nodes and phase cancellation, so a simple analogy can be used. In this case, waves created in a waterbed will be used to explain the effect.

Pressing on the end of a waterbed creates a "wave" of displacement that travels down the length of the bed until it reaches the opposite end and bounces back. The water pressure (stress) is the lowest, or negative with respect to the water at rest, at a point just in front of the wave and highest at a point just behind the wave. The pressures at the crest and in the trough are at the same pressure as the bed at rest. The wave will reflect back and forth until resistance to flow causes it to dampen out. The average pressure over time at any point in the bed will be exactly the same as the pressure at rest. Similarly, the average stress in a transformer off resonance will approach zero and there will be no net output.

Pressing on the end of the same bed repeatedly just after the wave has traveled down the length, reflected off the end, returned and reflected off the "driven" end will result in a standing wave. This means that one half of the bed is getting thicker as the other half is getting thinner and the center of the bed will be stationary. The center is the node and the thickness plotted over time of either end will form a sine wave. There will be no net pressure difference in the center, but the ends will have a pressure wave which form a sine wave 90^o out of phase with the displacement. The transformer again works in the same manner with no voltage at the node and an AC voltage at the ends. It is fairly simple to expand this concept to harmonics and to other resonate shapes.

Conclusion:

The number of different applications for piezoelectric ceramic, and in particular PZT ceramic, is too great to address in a single paper. The basic principals that have been set forth in this primer can, however, be used to both understand and design piezoelectric structures and devices. The ability to create devices of varying applications and shapes is greatly enhanced by the used of multilayer PZT ceramics.